

EHV AND UHV LINE LOADABILITY DEPENDENCE
ON VAR SUPPLY CAPABILITY

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ABSTRACT

Optimization algorithms for power system operation and planning employ EHV and UHV line loadability limits as well as var supply limits. Traditional line loadability limits are based on stability limits which assume infinite var supply capability. This paper shows the critical dependence of line loadability on var supplies and presents a method to compute line loadability limits that are consistent with var supply limits. As such the limiting quantities are suitable for optimization algorithms which utilize both line loadability and var supply constraints.

INTRODUCTION

Virtually all current and proposed optimization algorithms for power system operation and planning employ constraints on quantities such as line power flows and reactive power limits. In nonlinear optimization, these constraints are often enforced with due consideration for the interaction between constraints. That is, the var limits of generators are dependent on the variable real power generation being dispatched. Furthermore, global loadability limitations can be formulated into nonlinear algorithms. [6]. These later techniques do not, however, consider the affect of var limits on the Global loadability limits. Many recent methods have proposed single or iterative linear programming solutions [1-3]. The constraints imposed on these algorithms are frequently specified by single number limits on line flows, voltage levels, real and reactive power supply limits. For the line flow constraints, the justification for these single loadability limits is often based on composite ratings such as the St. Clair curves [4]. The interdependence of the line flow, voltage and power supply constraints was recently discussed and presented as a major area for further study [5]. In particular, the reactive power allocation was noted as a key factor in determining line loadability limits. In this paper, line loadability limits are shown to be critically related to var reserves. The importance of var allocation for voltage control has been well documented in the literature. In contrast, the importance of var reserves in establishing line loadabilities has not. The purpose of this paper is to illustrate the limitations of traditional loadability constraints and to propose the use of line loadability constraints that are consistent and directly dependent on reactive power limits.

LINE LOADABILITY

Consider the representation of figure 1. This equivalent was selected since the classical Sinusoidal stability limit applies only to elements which have "load independent" voltages at both ends. Furthermore, the classical sinusoidal stability limit has played a key role in the analytical development of St. Clair curves and stability margins [4].

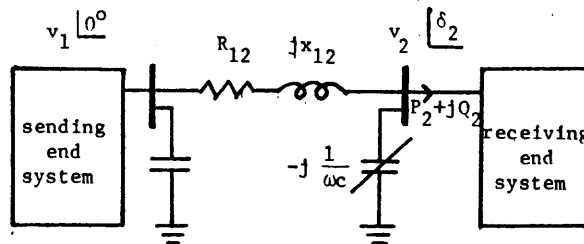


Figure 1.

Single line equivalent

The equations which describe the power flow across this line computed at bus 2 are:

$$P_2 = \frac{v_1 v_2}{|Z_{12}|} \cos(\delta_2 + \theta_{12}) - \frac{v_2^2}{|Z_{12}|} \cos(\theta_{12}) \quad (1)$$

$$Q_2 = \frac{v_1 v_2}{|Z_{12}|} \sin(\delta_2 + \theta_{12}) - \frac{v_2^2}{|Z_{12}|} \sin(\theta_{12}) + v_2^2 \omega c \quad (2)$$

where $Z_{12} = R_{12} + j x_{12} = |Z_{12}| \angle \theta_{12}$. If there is perfect voltage control at bus 2 (c is continuously variable and unlimited) then the maximum power transfer occurs when $\delta_2 = -\theta_{12}$. With perfect voltage control at both ends, the maximum loadability becomes dependent on this critical angle or on thermal limits. Traditionally, systems with short lines are considered to be thermally limited, and uncompensated longer line systems angle limited. Since the line impedance angle of the longer EHV and UHV lines approaches 90° the classical critical angle of -90° is clear from Equation (1). The use of stability margins is frequently based on this critical angle [4]. A 30% stability margin corresponding to $\delta_2 \approx -44^\circ$ is shown in Figure 2.

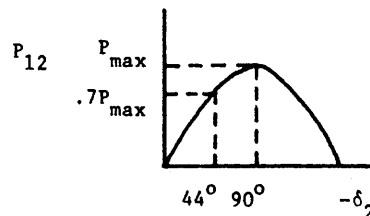


Figure 2.

Single line power transfer with perfect voltage control

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If the variable capacitor of Figure 1. is designed to control the power factor of the total load rather than voltage, the maximum power transfer is totally different. In this case, from Equation (2), with net power factor at 2 equal to 1,

$$v_1 \sin(\delta_2 + \theta_{12}) = v_2 \sin \theta_{12} \quad (3)$$

so that the real power transfer is

$$P_2 = \frac{v_1^2}{|Z_{12}| \sin \theta_{12}} \sin(2\delta_2 + 2\theta_{12}) - \frac{v_1^2 \cos \theta_{12}}{|Z_{12}| \sin^2 \theta_{12}} \sin^2(\delta_2 + \theta_{12}). \quad (4)$$

If there is perfect unity power factor control at bus 2 (c is continuously variable and unlimited) then the maximum power transfer occurs when $\delta_2 = -0.5 \theta_{12}$.

For long lines with θ_{12} near 90° , the critical angle is near -45° . Under these conditions, operation at -44° would have virtually zero stability margin. Since v_2 is not controlled however, the limiting constraint for this line could be voltage rather than angle. The point is that -44° cannot be used as a 30% stability margin for buses with power factor control rather than voltage control.

If the var source at bus 2 is limited (c is finite or fixed), the maximum power transfer criteria is again totally different. In this case, the jacobian of Equations (1) and (2) must be analyzed with c constant [6]. The jacobian is

$$\frac{\partial P_2}{\partial \delta_2} = -\frac{v_1 v_2}{|Z_{12}|} \sin(\delta_2 + \theta_{12}) \quad (5)$$

$$\frac{\partial P_2}{\partial v_2} = \frac{v_1}{|Z_{12}|} \cos(\delta_2 + \theta_{12}) - \frac{2v_2}{|Z_{12}|} \cos \theta_{12} \quad (6)$$

$$\frac{\partial Q_2}{\partial \delta_2} = \frac{v_1 v_2}{|Z_{12}|} \cos(\delta_2 + \theta_{12}) \quad (7)$$

$$\frac{\partial Q_2}{\partial v_2} = \frac{v_1}{|Z_{12}|} \sin(\delta_2 + \theta_{12}) - \frac{2v_2}{|Z_{12}|} (\sin \theta_{12} - |Z_{12}| \omega c). \quad (8)$$

or

$$J = \frac{1}{|Z_{12}|} \begin{bmatrix} -v_1 v_2 \sin(\delta_2 + \theta_{12}) & v_1 \cos(\delta_2 + \theta_{12}) - 2v_2 \cos \theta_{12} \\ v_1 v_2 \cos(\delta_2 + \theta_{12}) & v_1 \sin(\delta_2 + \theta_{12}) - 2v_2 (\sin \theta_{12} - |Z_{12}| \omega c) \end{bmatrix} \quad (9)$$

This matrix is singular when

$$2v_2 \cos \delta_2 (1 - |Z_{12}| \omega c) = v_1 \quad (10)$$

For lines with $\theta_{12} \approx 90^\circ$ $|Z_{12}| = x_{12}$ the affect of c on v_2 as a function of δ_2 is found from Equation (2) by substituting

$$Q_2 = P_2 \left(\frac{RPF}{PF} \right) = \frac{v_1 v_2}{x_{12}} \sin(-\delta_2) \left(\frac{RPF}{PF} \right), \quad (11)$$

where RPF and PF are the reactive power factor and power factor respectively. This substitution yields,

$$v_2 = \frac{v_1 \cos \delta_2 + v_1 \sin \delta_2 \left(\frac{RPF}{PF} \right)}{1 - x_{12} \omega c}. \quad (12)$$

Thus from Equation (10), the maximum transfer occurs when

$$\cos^2 \delta_2 + \cos \delta_2 \sin \delta_2 \left(\frac{RPF}{PF} \right) = 1/2. \quad (13)$$

An equivalent condition is found by manipulating Equation (13) to obtain,

$$A \cos(2\delta_2 - \phi) = 0. \quad (14)$$

where

$$A = \left(1 + \left(\frac{RPF}{PF} \right)^2 \right)^{1/2}$$

and

$$\phi = \tan^{-1} \left(\frac{RPF}{PF} \right). \quad (15)$$

For $P_{12} > 0$, the critical values of δ_2 are easily computed from Equations (14) and (15) as

$$\delta_2 = -\frac{\pi}{4} + \frac{\phi}{2}. \quad (16)$$

critical
const. c

It is important to note that Equations (14)-(16) are only functions of the power factor angle and not v_2 , x_{12} or c . Table 1. shows the critical values of δ_2 for several values of (RPF/PF) . The value of v_2 at each of these critical points would depend on c in accordance with Equation (12). The corresponding voltages at the point of maximum power transfer are also shown for various values of $x_{12} \omega c$. The significance of these figures can be seen by considering the following scenario.

Suppose that the shunt capacitance shown represents the upper limit of a discrete variable compensation bank designed to provide rated voltage when the line was loaded with $Q_2 = 0$ to an angle separation of -44° (chosen to correspond to a 30% stability margin). When the last stage is switched in, the bank is a fixed capacitance, and unity or lagging power factor loading beyond -45° is impossible. Thus the true stability margin was less than 2% rather than the presupposed 30%. Furthermore, the voltage at the critical power angle operating point is well within design objectives.

The St. Clair curves or their analytical equivalent have been proposed as constraints for maximum line and subsequent system loadability [8]. Their applicability hinges largely on var reserve capability. To illustrate this, consider a 400 mile 345 kv line with series

TABLE 1.

Single line critical angles with fixed v_1 and constant capacitive shunt compensation at bus 2

RPF/PF	PF	Critical δ_2	$x_{12}\omega c$	v_2
-0.6	.86 lead	-60.48°	0	1.0148
			0.1	1.1276
-0.3	.96 lead	-53.35°	0.1	0.93069
			0.2	1.0470
			0.3	1.1966
0.0	1.00	-45.00°	0.2	0.8838
			0.3	1.0102
			0.4	1.1785
			0.5	1.2464
+0.3	.96 lag	-36.65°	0.3	0.8903
			0.4	1.0387
			0.5	1.2464
+0.7	.82 lag	-27.50°	0.3	0.8053
			0.4	0.9395
			0.5	1.1274
+1.4	.58 lag	-17.77°	0.4	0.8751
			0.5	1.0501
			0.6	1.3126

impedance of $0.023 + j0.25$ p.u. and receiving end line charging of $B = 1.32$ p.u. [4]. If this line has perfect voltage control at the sending end, and limited control at the receiving end, the maximum loadability can be determined using the preceding analysis. If line charging alone provides the voltage support at the receiving end, the maximum unity power factor load which can be delivered (neglecting line resistance) is found from equations (10) and (12) to be 2.98 p.u. on a 100mw base. This maximum occurs at an angle separation of -45° with $v_2 = 1.055$. The maximum loadability specified by the St. Clair curves for this line is 2.56 p.u. (0.8SIL) [4]. The latter is 86% of the maximum loadability, or would be considered to have a 14% stability margin rather than the 35% as originally intended by the St. Clair curves. If a 35% stability margin is to be met, the receiving bus must have additional var reserves even though they are not needed for operation at the "maximum" loadings of 2.56 p.u. or 2.98 p.u.

AFFECTS OF VAR SOURCE MODELING IN COMPUTER APPLICATIONS

The mathematical modeling of var sources has a significant affect on the critical angles at maximum loadability. When a capacitor bank operates at values near rated voltage, the bank may in some applications be considered a constant reactive power source. The relationships of the previous section can be used to illustrate these affects. Consider a single line with $Z_{12} = j0.1$ and perfect voltage control at bus 1. If a variable capacitor bank with maximum setting of $\omega c = 3.0$ is used as limited voltage control at bus 2, the maximum power transfer occurs when $\delta_2 = -45^\circ$, $v_2 = 1.0102$, and $P_2 = 7.1435$. The reactive power output of the capacitor bank at this maximum condition is 3.0615. Alternatively, if the problem is restated to include a variable var source with $Q_{max} = 3.0615$, the maximum power transfer condition is different. In this case, there is no shunt compensation, and the maximum power transfer will occur when the net injected reactive power at bus 2 is 3.0615. The maximum real power which can be transferred under

these conditions is $P_2 = 7.458$, which occurs at a critical angle separation of $\delta_2 = -56^\circ$, and at a voltage of $v_2 = 0.898$. Thus the var supply at both critical points is the same, but the operating points are considerably different. A relationship exists between the maximum var capability of a controller and the critical power angle. This relationship is presented below.

CRITICAL ANGLE DETERMINATION WITH FINITE VAR SUPPLY CAPABILITY

Consider the addition of a synchronous condenser at bus two of Figure 1, with maximum var capability of Q_{max} . Suppose also that the desired voltage is $v_2 = v_1$, and c is some fixed capacitance (line charging or additional compensation). With this perfect voltage control, the critical angle δ_2 is equal to $-\theta_{12}$, provided Q_{max} is large enough to allow the angle separation. If Q_{max} is not large enough to allow the separation to reach $-\theta_{12}$, the angle δ_2 at the instant the limit Q_{max} is hit is found from,

$$Q_2 = \frac{v_1^2}{|Z_{12}|} \sin(\hat{\delta}_2 + \theta_{12}) - \frac{v_1^2}{|Z_{12}|} \sin \theta_{12} + v_1^2 \omega c + Q_{max}. \quad (17)$$

For simplicity, consider the case where $Q_2 = 0$ (unity power factor load) and $\theta_{12} = 90^\circ$, $|Z_{12}| = x_{12}$ (lossless line). The angle $\hat{\delta}_2$ at the instant the limit Q_{max} is hit is,

$$\hat{\delta}_2 = -\cos^{-1} \left[1 - x_{12}\omega c - \frac{x_{12}}{v_1^2} Q_{max} \right]. \quad (18)$$

This corresponds to a real power load less than v_1^2/x_{12} . For increases in real power load, the synchronous condenser can be considered a constant var source with output equal to Q_{max} . Under these conditions, the system may or may not be able to supply additional real power load. The voltage v_2 would be found from

$$0 = \frac{v_1 v_2}{x_{12}} \cos \delta_2 - \frac{v_2^2}{x_{12}} [1 - x_{12}\omega c] + Q_{max}, \quad (19)$$

or

$$v_2 = \frac{1}{2(1-x_{12}\omega c)} \left[v_1 \cos \delta_2 + \sqrt{v_1^2 \cos^2 \delta_2 + 4Q_{max} x_{12} (1-x_{12}\omega c)} \right] \quad (20)$$

With an uncontrolled bus 2, the critical δ_2 is found from Equation (10) evaluated at v_2 from Equation (20),

$$\cos \delta_2 [v_1 \cos \delta_2 + \sqrt{v_1^2 \cos^2 \delta_2 + 4Q_{max} x_{12} (1-x_{12}\omega c)}] = v_1. \quad (21)$$

A solution is

$$\delta_2^* = -\cos^{-1} \left[\frac{v_1}{\sqrt{2v_1^2 + 4Q_{max} x_{12} (1-x_{12}\omega c)}} \right]. \quad (22)$$

For a given set of v_1 , x_{12} , ωc , and Q_{max} , the critical power transfer angle will be given by δ_2 or δ_2^* . Examples of each are given in Table 2. The following criteria establishes the critical angle displacement across a

line with fixed v_1 and continuously variable but finite var compensation at bus 2,

$$\delta_2 = \begin{cases} -90^\circ & \text{If } Q_{\max} > v_1^2 \left(\frac{1}{x_{12}} - \omega c \right) \\ -\max[|\hat{\delta}_2|, |\delta_2^*|] & \text{otherwise.} \end{cases} \quad (23)$$

TABLE 2.

Single line critical angles with fixed v_1 and continuously variable but finite var compensation at bus 2.

(a)	(b)
$x_{12} = 0.1$ p.u.	$x_{12} = 0.1$ p.u.
$\omega c = 1.0$ p.u.	$\omega c = 3.0$ p.u.
$Q_{\max} = 2.0$ p.u.	$Q_{\max} = 2.0$ p.u.
$v_1 = 1.0$ p.u.	$v_1 = 1.0$ p.u.
$\hat{\delta}_2 = -46^\circ$	$\hat{\delta}_2 = -60^\circ$
$\delta_2^* = -53^\circ$	$\delta_2^* = -51^\circ$
$\delta_2 = -53^\circ$ (beyond var limit critical operating point)	$\delta_2 = -60^\circ$ (at var limit operating point)
$v_2 = 1.0$ at Q limit	$v_2 = 1.0$ at Q limit
$v_2 = 0.912$ at δ_2 critical	$v_2 = 1.0$ at δ_2 critical
$p_2 = 7.19$ at Q limit	$p_2 = 8.66$ at Q limit
$p_2 = 7.28$ at δ_2 critical	$p_2 = 8.66$ at δ_2 critical

The physical implications of the preceding analysis are that var limits must be great enough to allow operation at an angle displacement of -90° even though normal operation will be limited to -44° (thus a true 30% stability margin). If var capabilities are specified to control voltage only to a displacement of say -44° , the true stability margin may be zero.

MINIMUM VAR SUPPLY CAPABILITY

In the preceding section, the affects of limited var supply on power transfer and stability margin computation were presented. The maximum var capability is critical in the determination of maximum power transfer and critical angular separation. For a given operating point, the var reserves influence the stability margin. Consider the following definition of stability margin for the single line system of Figure 1.,

$$\text{s.m.} = \frac{\Delta}{\text{crit}} \left(1 - \frac{p_2^0}{p_2} \right) 100 \quad (24)$$

where p_2^0 is some operating point power, and p_2 is the maximum value of that power for which steady state operation is possible. If a voltage control device is to be added at bus 2, the maximum var capability of that device will largely determine the operating point voltage and the operating point stability margin. An

algorithm for selecting the minimum value of var reserves necessary to maintain acceptable voltage and specified stability margin is presented below.

For this analysis, consider the lossless line ($\theta_{12} = 90^\circ$, $|Z_{12}| = x_{12}$) and unity power factor load ($Q_2 = 0$). Furthermore let the desired controlled voltage at bus 2 be equal to the fixed bus 1 voltage, ($v_2 = v_1$) so that the problem is stated as follows:

- Given: a) An operating point load P_2^0
 b) A minimum acceptable voltage v_2
 c) A desired value of s.m.
 des
 d) No lower bound on injected Q
 e) Values for c_1 , x_{12} , v_1 ($x_{12} \omega c < 1$)

- Find: a) The minimum value of reactive power required into bus 2 (min Q_{\max}^0) to ensure $v_2^0 \geq v_2$,
 $v_2^0 \geq v_2$ and $\text{s.m.}^0 \geq \text{s.m.}$
 b) The operating point v_2^0 , δ_2^0 , Q^0
 c) The operating point corresponding to full utilization of desired stability margin,
 v_2^{max} , δ_2^{max} , P_2^{max}
 d) The operating point corresponding to maximum power transfer v_2^{crit} , δ_2^{crit} , P_2^{crit}

Solution:

Step # 1 test: If $P_2^0 > \frac{v_1^2}{x_{12}}$ there is no solution (stop)

If $P_2^0 \leq \frac{v_1^2}{x_{12}}$ there may be a solution (go to step # 2)

Step # 2 compute: $P_2^{\text{max}} = P_2^0 \left(\frac{100}{100 - \text{s.m.}_{\text{des}}} \right)$

test: If $P_2^{\text{max}} > \frac{v_1^2}{x_{12}}$ there is no solution with $\text{s.m.}^0 \geq \text{s.m.}_{\text{des}}$ (go to step # 3)

If $P_2^{\text{max}} \leq \frac{v_1^2}{x_{12}}$ there is a solution with $-\delta_2^{\text{crit}} < 90^\circ$ and $\text{s.m.}^0 \geq \text{s.m.}_{\text{des}}$ (go to step # 4)

Step # 3 compute: $v_2^0 = v_2^{\text{max}} = v_2^{\text{crit}} = v_1$

$$\delta_2^0 = \sin^{-1} \left(\frac{P_2^0 x_{12}}{v_1^2} \right)$$

$$Q^0 = \frac{v_1^2}{x_{12}} (1 - x_{12} \omega c - \cos \delta_2^0)$$

$$\delta_2^{\text{max}} = \delta_2^{\text{crit}} = -90^\circ$$

$$P_2^{\text{crit}} = \frac{v_1^2}{x_{12}}$$

$\text{s.m.}^0 =$ found from Equation (24) $\leq \text{s.m.}_{\text{des}}$

$$\min Q_{\max} = \frac{v_1^2}{x_{12}} (1 - x_{12} \omega c)$$

(stop)

Step #4 compute: $v_2 = \left(\frac{P_2^2 x_{12}^2}{v_1^2} + \frac{v_1^2}{4(1-x_{12}\omega c)^2} \right)^{1/2}$

cfit
 s.m.

(This is the critical voltage if the line is limited by stability margin. This equation was found by solving Equations (1) and (10) with $P_2 = P_2^{\max}$)

test: If $v_2 \geq v_1$ then the line is stability

cfit
 s.m.

limited, and the critical operating point will be exactly when the loading reaches P_2^{\max} with $v_2 = v_1$.

cfit (go to step # 5)

If $v_2 < v_1$ then the line is stability

cfit
 s.m.

limited, and the critical operating point will be exactly when the loading reaches P_2^{\max} with $v_2 = v_1$.

cfit (go to step # 6)

If $v_2 < v_2$, then the line is voltage limited.

cfit
 s.m.

so $v_2^{\max} = v_2$ (go to step # 7)

Step #5 compute: $v_2^0 = v_2^{\max} = v_2^{\text{cfit}} = v_1$

$$\delta_2^0 = -\sin^{-1} \left(\frac{P_2^0 x_{12}}{v_1} \right)$$

$$Q^0 = \frac{v_1^2}{x_{12}} (1 - x_{12} \omega c - \cos \delta_2^0)$$

$$\delta_2^{\max} = \delta_2^{\text{cfit}} = -\sin^{-1} \left(\frac{P_2^{\max} x_{12}}{v_1} \right)$$

$$P_2^{\text{cfit}} = P_2^{\max}$$

$$\text{s.m.}^0 = \text{s.m.}_{\text{des}}$$

$$\min Q_{\max} = \frac{v_1^2}{x_{12}} (1 - x_{12} \omega c - \cos \delta_2^{\text{cfit}})$$

(stop)

Step #6 compute: $v_2^{\max} = v_2^{\text{cfit}} = v_2^{\text{s.m.}}$

$$\delta_2^{\max} = \delta_2^{\text{cfit}} = -\sin^{-1} \left(\frac{P_2^{\max} x_{12}}{v_2^{\max} v_1} \right)$$

$$P_2^{\text{cfit}} = P_2^{\max}$$

$$\text{s.m.}^0 = \text{s.m.}_{\text{des}}$$

$$\min Q_{\max} = \frac{v_2^{\max} v_1}{x_{12}} (1 - x_{12} \omega c) - \frac{v_1 v_2^{\max}}{x_{12}} \cos \delta_2^{\max}$$

(go to step #8)

Step #7 compute: $v_2^{\max} = v_2$

$$\delta_2^{\max} = -\sin^{-1} \left(\frac{P_2^{\max} x_{12}}{v_2 v_1} \right)$$

$$\min Q_{\max} = \frac{v_2^2}{x_{12}} (1 - x_{12} \omega c) - \frac{v_1 v_2}{x_{12}} \cos \delta_2^{\max}$$

δ_2^{cfit} = found from Equation (22) evaluated at $\min Q_{\max}$

v_2^{cfit} = found from Equation (20) evaluated at δ_2^{cfit} and $\min Q_{\max}$

P_2^{cfit} = found from Equation (1) evaluated at δ_2^{cfit} and v_2^{cfit}

s.m.^0 = found from Equation (24)

(go to step # 8)

Step #8 The initial operating point may be

$$v_2^0 = 1 \text{ or } v_2^0 < 1$$

compute: $\delta_2^0 = -\sin^{-1} \left(\frac{P_2^0 x_{12}}{v_1} \right)$

Req

$$Q^0 = \frac{v_1^2}{x_{12}} (1 - x_{12} \omega c - \cos \delta_2^0)$$

Req

test: If $Q_{\text{Req}}^0 \leq \min Q_{\max}$ the initial point is

$$v_2^0 = v_1$$

(go to step # 9)

If $Q_{\text{Req}}^0 > \min Q_{\max}$ the initial point is

$$v_2^0 < v_1$$

(go to step # 10)

Step #9 compute: $v_2^0 = v_1$

$$\delta_2^0 = -\sin^{-1} \left(\frac{P_2^0 x_{12}}{v_1} \right)$$

$$Q^0 = Q_{\text{Req}}^0$$

(stop)

Step #10 compute: δ_2^0 = found from Equations (A-1) to (A-10) (see Appendix A)

$$v_2^0 = \frac{x_{12} P_2^0}{v_1 \sin(-\delta_2^0)}$$

$$Q^0 = \min Q_{\max}$$

(stop)

The above ten step algorithm has been applied to an example single line system. The results for various parameters are given in Table 3.

Table 3.
min Q_{\max} determination

Specified Parameters							Solution								
P_2^0	v_2	s.m. des	x_{12}	ωc	v_1	δ_2^0	v_2^0	Q^0	δ_2^{\max}	v_2^{\max}	δ_2^{crit}	v_2^{crit}	P_2^{crit}	s.m. ⁰	min Q_{\max}
3.0	0.95	30%	0.1	0	1.00	-17 ⁰	1.00	0.46	-27 ⁰	0.95	-48 ⁰	0.74	5.52	46%	0.54
3.0	0.90	30%	0.1	0	1.00	-18 ⁰	0.97	0.19	-28 ⁰	0.90	-46 ⁰	0.72	5.18	42%	0.19
4.0	0.95	30%	0.1	0	1.00	-24 ⁰	1.00	0.83	-37 ⁰	0.95	-51 ⁰	0.80	6.27	36%	1.44
4.0	0.90	30%	0.1	1.0	1.00	-24 ⁰	1.00	-0.16	-39 ⁰	0.90	-47 ⁰	0.81	5.88	32%	0.34
5.0	0.95	30%	0.1	0	1.00	-30 ⁰	1.00	1.34	-49 ⁰	0.95	-55 ⁰	0.89	7.24	31%	2.76
5.0	0.90	30%	0.1	1.0	1.00	-30 ⁰	1.00	0.34	-52 ⁰	0.905	-52 ⁰	0.905	7.14	30%	1.81
6.0	0.95	30%	0.1	0	1.00	-37 ⁰	1.00	2.00	-60 ⁰	0.99	-60 ⁰	0.99	8.57	30%	4.85
6.0	0.95	30%	0.1	2.0	1.00	-37 ⁰	1.00	0.00	-59 ⁰	1.00	-59 ⁰	1.00	8.57	30%	2.85
7.0	0.95	30%	0.1	0	1.00	-44 ⁰	1.00	2.86	-90 ⁰	1.00	-90 ⁰	1.00	10.00	30%	10.00
8.0	0.95	30%	0.1	0	1.00	-53 ⁰	1.00	4.00	-90 ⁰	1.00	-90 ⁰	1.00	10.00	20%	10.00

SYSTEM APPLICATIONS

System studies utilizing exact load flow studies can accurately model finite var supply capabilities and their affects on the steady state solution. The linear load flow techniques utilized in the computation of simultaneous interchange and maximum system loadability however cannot account for voltage control with finite var reserves, and thus depend on line loading or angle constraints to insure adequate stability margins. Reactive power limits can be used with linear constraints to insure adequate voltage control, however the analysis presented earlier clearly shows that the critical loading may easily occur at the limit of the var source. If the var limit is not based on utilization at an angle consistent with the line loading or angle constraint, the maximum may not satisfy the stability margin implicitly incorporated in the St. Clair loadability limit. The applicability of the St. Clair curve can be tested by computing the minimum var requirements in accordance with the ten step algorithm presented above. Consistency would be maintained by specifying P_2^0 equal to the loadability given by the St. Clair curve, $s.m._{\text{des}} = 35\%$ and $v_2 = 0.95$. If the resulting minimum is less than the var capability at bus 2, then P_2^0 can be utilized as a line loadability limit. If the resulting minimum is greater than the var capability at bus 2, then the line loadability limit is less than P_2^0 , and should be computed based on a critical angle separation less than 90⁰ (Equations 20-23).

MULTIPLE FINITE VAR SOURCES

The examples shown above illustrate the sensitivity of the line loadability to var reserves. The analysis also demonstrated a significant difference between modeling var source limits as reactive power limits and capacitive limits. This is due to the strong coupling and dependence between the voltage sensitivity and the var support. These phenomena were illustrated for the single line with perfect voltage control at one end. Single lines were considered since one objective of the analysis was to illustrate that the single line loadabilities given by the St. Clair curves and recent work of reference [4] specifically address the single line case. The above results indicate that these traditional loadabilities should be used with considerable caution and indeed revised when the required var support does not exist.

The extension of the analysis presented in this paper to multiple line and multiple finite var sources requires the concept of system loadability. System loadability as presented in reference [3] utilizes single line limits and thus assumes that the single line stability margins provide sufficient constraints on the global system stability margin. Under this assumption, the consideration of finite var support at one end of a single line is a first step towards formulating the more general solution. The results presented in references [6] and [7] define the system loadability in a more global sense and formulate feasibility regions. These feasibility regions consider multiple lines with either infinite var support or fixed var supply. As such they compute system stability margins assuming either perfect voltage control or no voltage control. The existence of limited voltage control due to finite var supplies would have a significant affect on the global feasibility region and resulting system stability margins. Extension of the results presented in this paper to the more general case of multiple finite var supplies would require the analysis of multiple intersecting feasibility regions. Based on the results of this paper, it is reasonable to conjecture that the intersection may very well pass directly through the boundary defined by the var limits.

CONCLUSIONS

The importance of var supplies in maintaining acceptable voltage levels is well known, and widely studied. The importance of var reserves in maintaining stability margins has not received equal emphasis in the literature. In fact, virtually all work in the area of maximum power transfer and import capability is based on the assumption of unlimited var reserves.

The primary purposes of this paper were to call attention to the importance of var reserve modeling in computer applications and to form the basis for the analysis of interconnected systems. Specifically, the following points were made: (a) The critical angular displacement across lines with finite var supplies may be substantially less than 90⁰. Indeed realistic cases where maximum power transfer occurs at less than 30⁰ with rated voltage at both ends were shown. (b) There is a significant difference between modeling var limits as fixed capacitors and as fixed reactive power sources. This is particularly important when switched capacitor banks are modeled as PV buses. (c) Var reserves must be capable of maintaining voltage well beyond normal

operating conditions. That is, if a stability margin is desired, the minimum var reserves must be greater than the var requirements at the top end of the stability margin. (d) Stability limited line constraints based on St. Clair curves or their analytic equivalent apply only to lines with var reserves large enough to permit loading to an angular displacement of 90° . (e) The maximum power transfer across a line with limited voltage control may very likely occur exactly at the point the var limit is reached regardless of the angular displacement.

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References

- [1] G. L. Landgren, S. W. Anderson, "Simultaneous Power Interchange Capability Analysis," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-92, 1973, pp. 1973-1986.
- [2] B. Stott, J. L. Marinho, "Linear Programming for Power-System Network Security Applications," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, 1979, pp. 837-848.
- [3] L. L. Garver, P. R. Van Horne and K. A. Wirgau, "Load Supplying Capability of Generation-Transmission Networks," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, 1979, pp. 957-962.
- [4] R. D. Dunlop, R. Gutman and P. P. Marchenko, "Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, 1979, pp. 606-617.
- [5] W. A. Johnson et al., "EHV Operating Problems Associated with Reactive Control," Paper 80 SM 513-2, presented at the IEEE PES 1980 summer meeting, Minneapolis, Minn., July 13-18, 1980, pp. 1-6.
- [6] F. D. Galiana, "Power Voltage Limitations Imposed by the Network Structure of a Power System," Proceedings of the 1975 Power Industry Computer Applications (PICA) Conference, June 1975, pp. 355-363.
- [7] C. J. Tavora, O. J. M. Smith, "Equilibrium Analysis of Power Systems," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-91, No. 3, May/June 1972, pp. 1131-1137.
- [8] Discussion and closure to Reference [3].

Appendix A

The following derivation solves the following power flow equations for the nominal values of v_2^0 and δ_2^0 for fixed v_1 , x_{12} , ω , c and $\min Q_{\max}$.

$$P_2^0 = \frac{v_1 v_2^0}{x_{12}} \sin(-\delta_2^0) \quad (A-1)$$

$$\begin{aligned} \min Q_{\max} &= \frac{v_1 v_2^0}{x_{12}} \cos(-\delta_2^0) - \frac{v_2^{0^2}}{x_{12}} [1 - x_{12} \omega c] \\ &+ \min Q_{\max} \end{aligned} \quad (A-2)$$

From Equation (A-1)

$$v_2^0 = \frac{x_{12} P_2^0}{v_1 \sin(-\delta_2^0)} \quad (A-3)$$

Substitution into Equation (A-2) yields

$$\min Q_{\max} + \frac{P_2^0 \cos(-\delta_2^0)}{\sin(-\delta_2^0)} - \frac{x_{12} P_2^{0^2} (1 - x_{12} \omega c)}{v_1^2 \sin^2(-\delta_2^0)} \quad (A-4)$$

Multiplication by $v_1^2 \sin^2(-\delta_2^0)$ and the use of double angle identities gives

$$\begin{aligned} &(\min Q_{\max} v_1^2) \cos 2\delta_2^0 - (P_2^0 v_1^2) \sin(-2\delta_2^0) \\ &- \min Q_{\max} v_1^2 + 2x_{12} P_2^{0^2} (1 - x_{12} \omega c) = 0 \end{aligned} \quad (A-5)$$

This equation can be written as

$$C_1 \cos(-2\delta_2^0) + C_2 = C_3 \quad (A-6)$$

where

$$C_1 = v_1 \sqrt{(\min Q_{\max})^2 + P_2^{0^2}} \quad (A-7)$$

$$C_2 = \tan^{-1} \frac{P_2^0}{\min Q_{\max}} \quad (A-8)$$

$$C_3 = \min Q_{\max} v_1^2 - 2x_{12} P_2^{0^2} (1 - x_{12} \omega c) \quad (A-9)$$

so that the solution is

$$\delta_2^0 = \frac{1}{2} (C_2 - \cos^{-1} \frac{C_3}{C_1}) \quad (A-10)$$

and v_2^0 is found from Equation (A-3).

Discussion

M. Winokur (Imperial College, London, England): The authors are to be congratulated for their well presented and comprehensive paper on line loadability constraints dependence on var reserves. This subject has not received much emphasis in the literature, as the authors rightly point out, although some work has recently emerged (A) - (C).

A point which we feel merits some discussion is the choice of δ_{crit} in the case of static compensation combined with a controllable var source. The authors find the angle δ_2 at the instant the limit Q_{max} is hit from (18) and the angle δ_2^* from (22) for maximum transmissible real power after Q_{max} is reached. In (23), for $Q_{max} < V_1^2 (1/X_{12} - \omega c)$, the critical angle is chosen as

$$\delta_{crit} = -\max\{\delta_2, \delta_2^*\}$$

However if P_{crit} is defined as the maximum value of power transfer for which steady state operation is possible then one should always define

$$\delta_{crit} = \delta_2^*$$

Specifically in the case of the example (b) of Table 2 if:

$$\delta_{crit} = \delta_2^* = 51^\circ$$

the resulting power transfer and voltage at this stability limit point are:

$$P_{crit} = 8.92 \text{ p.u.}$$

$$V_{crit} = 1.14 \text{ p.u.}$$

What happens is that because of the value of the static compensation after the var source has reached its limit, the power demand can still increase causing a rise in load voltage and a corresponding reduction in the angle across the line.

In the 8th line of Table 3 the same occurs resulting in

$$\delta_{crit} = 54^\circ$$

$$P_{crit} = 8.64 \text{ p.u.}$$

$$V_{crit} = 1.07 \text{ p.u.}$$

$$s.m.o. = 31\%$$

If this proposed criteria of $\delta_{crit} = \delta_2^*$ is applied step No. 5 of the algorithm needs to be slightly modified resulting in:

$$V_{crit} > V_{2max}; \delta_{crit} < \delta_{2max}; P_{crit} > P_{2max}; s.m.o. > s.m.o.$$

It is interesting to note that the values of the critical load voltage can be very close to, or even higher than the sending end voltage, as shown in tables 1 and 3. Hence the commonly used criteria of voltage drop larger than a given amount, say 10% [B], to indicate critical operation could be very misleading. Do the authors think that a comparison between the angle across the line and the critical angle would be a better indication of stability? If so, how can δ_{crit} be calculated given some static compensation and var source limit for loads with power factor different from unity?

REFERENCES

- [A] C. Barbier, J. P. Barret, "An Analysis of phenomena of voltage collapse on a transmission system", *RGIE*, Vol. 89, part 10, Oct. 1980, pp. 672-690.
- [B] W. R. Lachs, "Voltage collapse in EHV power systems", paper A78 057-2, presented at the IEEE PES Winter Meeting, New York, Jan. 29 - Feb. 3, 1978.
- [C] W. R. Lachs, "Reactive power control in system emergencies", *Proceedings of the IEE International Conf. on Power System Monitoring and Control*, London, June 1980, pp. 149-153.

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T. W. Kay, P. W. Sauer, R. D. Shultz and R. A. Smith: The discussor has brought up several interesting points about the paper. In order to address the questions, it is necessary to first comment on the meaning of a fixed Q_{max} supply. The limits on voltage control devices are normally based on the limiting output of a voltage regulator. When voltage is allowed to vary, these limits do not normally translate into fixed reactive power. The comment by M. Winokur that the critical angle and power for example (b) of Table 2 should be -51° and 8.92 pu is not clear to us. This would have a load voltage of 1.14 pu which would normally initiate regulator action to reduce the voltage level and consequently the var output. Thus if the regulator set point is 1.0 pu v., the operating point of 8.92 pu power is not feasible. His comment brings up the interesting question of the affect of the regulator voltage set point on maximum power transfer. His observation that critical operating points can occur with seemingly "good" voltages is exactly correct and was part of our motivation for studying this problem.

We are currently replacing the use of "fixed var" supplies with more realistic models of voltage control devices. These include var supplies with physical limits other than fixed Q_{max} . The papers by Lachs consistently point to tap changing under load transformers as a possible cause of voltage collapse. We are currently studying the static and dynamic affects of TCUL action on steady state feasibility and dynamic stability. We would like to thank M. Winokur for his interest in the paper and his valuable comments. We agree with him that this area has been neglected for many years and needs considerable further investigation.

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